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A REVIEW OF PROFESSOR MOORE'S PRESIDENTIAL ADDRESS[1]

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On December 29, 1902, Professor E. H. Moore of the University of Chicago delivered the Presidential Address before the American Mathematical Society, "On the Foundations of Mathematics." It is with great hesitation that I make any attempt to consider this epoch-making pronouncement of the views and visions, ideals and hopes, of this noted mathematician and general thinker as regards mathematical content and methods.

To review Professor Moore's paper as it should be reviewed would require more time than the few minutes which are allotted me, and even if there were ample time, I am quite sure that my best efforts would fall far short of even adequate justice to its far-reaching influence. For this reason and because of the present day conditions in our mathematical world I should prefer to have you regard this paper as merely a stimulus to re-read Professor Moore's address with the utmost care, filling in for yourselves those thoughts which will naturally arise because of your own peculiar environment and point of view.

Merely for the sake of clarity I shall divide my remarks into the following general headings:

A. Preliminary Statements.
B. The General Settings in the Years 1901, 1902, 1903.
C. Desirable Changes Advocated by Professor Moore.
D. Goals Set Up That Have Not Been Achieved, OR Proposals as to Which There is Still Controversy.

Necessarily there will be difference of opinion as to the amount of emphasis to be placed on the word "desirable." Necessarily also questions will at once arise as to the possibility and practicability of desirable changes, the rapidity with which they can be made and the various modes and methods of making them.

[1] Read at the meeting of the National Council of Teachers of Mathematics, Feb. 20, 1926, Washington, D. C.

A. Preliminary Statements

The first part of Professor Moore's paper concerns itself with a brief general discussion of abstract mathematics on the one hand and pure and applied mathematics on the other. Keynotes are pointed out which are afterwards applied to elementary mathematics as follows:

1. Abstract mathematics involves the use of a definite system of undefined symbols in conjunction with a definite system of postulates. Its symbolism gives power, rigor and simplicity of procedure but, of course, does not obviate essential difficulties in a scientific, logical analysis.

2. All science, logic and mathematics included, is not fixed by metes and bounds, but in ideals as well as in achievements is a function of the epoch.

3. The division into deductive and inductive sciences cannot be wholly maintained and is understood in its entirety by the devotee of science only when he has reached a considerable degree of maturity.

4. The attitude of the practical mathematician towards the abstract mathematics and especially its symbolism, is akin to that of the practical physicist toward theoretical mathematics, the practical engineer toward theoretical mathematics, the practical engineer toward theoretical physics of mathematics.

5. Pure mathematics in its ordinary acceptation may be taken as the systematic development of the properties of the fundamental mathematical notions of number and form which were among the earliest to enter into the formulas of science, these formulas being more or less exact descriptions of phenomena.

6. Arithmetic and geometry closely united in mensuration and trigonometry early reached a high degree of advancement, but the development of the generalized literal notation of algebra largely in response to the demands of mechanics, astronomy and physics in the 17th century bound arithmetic and geometry still more closely together into analytic geometry and calculus, those mighty instruments of research which were developed further and further in the 18th and 19th centuries.

7. Especially during the 19th century came the critical reorganization and development of the foundations of pure mathematics (i.e., 'the mathematics of precision') independent of ap-

plied mathematics (i.e., 'the mathematics of approximation'), causing a gap towards the bridging of which agencies are at work.

Such is the gist of the preliminary statements made by Professor Moore on December 29, 1902.

B. The General Settings in the Years 1901, 1902, 1903

1. John Perry in 1901 at the first session in Glasgow of the Section on Education of the British Association in connection with the section on Mathematics and Physics delivered a paper on "The Teaching of Mathematics" emphasizing strongly the necessity for emancipation from Euclid as the sole authority in geometry especially with reference to sequence of propositions, and laying emphasis on practical mathematics based upon the notions underlying the infinitesimal calculus taken as axioms, map drawing to a scale, the use of squared paper, the use of decimals as numbers, etc.

He laid down as an absolute pre-requisite that the boy should be made familiar by every possible way (experiment, illustration, measurement, etc.) with the ideas to which he applies his logic and should be thoroughly interested in the subject studied.

2. This effort of Perry was part of a movement already in existence in England for thirty years designed to relieve secondary teachers from the burden of a too precise examination system imposed by the great examining bodies, and was followed in England by the appointment of a strong committee under the chairmanship of Professor Forsyth to report upon improvement in the teaching of mathematics, especially elementary mathematics, and means of accomplishment. This report was distinctly favorable, though in a conservative way, and initial changes were started.

3. In the United States the interest in the Perry movement was widespread and intense, and the years 1901, 1902, 1903 were notable because of the almost spontaneous formation of associations of teachers of mathematics for the vital improvement of mathematical teaching, and though necessarily they differed as to content, methods of approach, points of emphasis, sequence of topics, etc., all were animated by an earnest desire for the true development of the flesh and blood child with due regard for his personal equation.

Among the associations active in the consideration of the Perry Movement may be mentioned: The New England Association of Teachers of Mathematics; The Central Association of Science and Mathematics Teachers, with headquarters in Chicago, which was enthusiastic for the Perry Movement; The Association of Teachers of Mathematics in the Middle States and Maryland (including the District of Columbia) which was more conservative.

The Association of Teachers of Mathematics in the Middle States and Maryland was in the process of formation during the year 1903 and had its first meeting in New York City, November 28, 1903. At that meeting the present writer read the first paper presented to the Association on "The Laboratory Method of Teaching Mathematics," setting forth the laboratory method which had been in use in the schools of Washington, D. C., for many years, the suggestive plan of the Central Association of Science and Mathematics Teachers, and the general conditions obtaining at that time. Questions that are vital in 1926 were vital in 1903 and many desirable changes now advocated were then advocated by many.

This brings us to the consideration of that part of Professor Moore's address dealing with changes, bearing in mind:

(1) That Professor Moore spoke from the standpoint of the pure mathematician following very closely many of the ideas advanced by Professor Perry.

(2) That the International Commission on the Teaching of Mathematics after a comprehensive and exhaustive investigation as to facts and conclusions, issued a series of reports in 1911, the most vital and interesting of which from our immediate point of view being those of the three American Commissioners, Professor Smith, Professor Osgood, and Professor Young.

(3) That the World War of 1914 while interrupting the work of the Commission created new fields of investigations, and methods of dealing with them which has profoundly affected all educational problems especially those pertaining to mathematics.

(4) That the Junior High Movement was not in the vision of Professor Moore.

C. Desirable Changes Advocated by Professor Moore

1. Diminished emphasis on the systematic and formal side, and increased emphasis on the practical side (e.g., on arithmetic computation, elements of mechanical drawing, graphic methods, etc.) by connecting abstract mathematics with subjects which are naturally of interest to the boy and by using [1]those theorectical processes that may be checked by laboratory processes, so as to give to the very young students in their impressionable years in a thoroughly concrete and captivating form, in addition to trigonometry, the wonderful new notions of the 17th century, the essentials of analytics and calculus, and the practical use of the first mathematical tools in such a way that they will be interested in the theory of the tools themselves and so regard mathematics as a fundamental reality of the domain of thought and not as a collection of symbols and arbitrary rules.

2. It is pedagogically sound at first to take a large body of basal principles, reserving for later years a more philosophical treatment starting with a smaller number of such propositions.

3. As regards elementary schools: The kindergarten methods should be extended into the grades, but not to such an extent as to prevent appreciation of the abstract methods that come later; and the pupils should be trained in powers of observation, experimentation, reflection and deduction by direct connection with matters of a thoroughly concrete character (e.g., drawing, paper folding, elementary mechanical drawing, construction of models, graphical representations of one magnitude depending on another and their concrete meanings, use of co-ordinate paper) thus enriching and vitalizing the materials and methods, leading to the study of intuitive geometry and linking geometry with numerical and literal arithmetic.

4. As regards secondary schools: There should be no water-tight compartments. The systematic development of theoretical physics and of theoretical mathematics should come later. For the benefit of the student, at first the physics should be practical and likewise in geometry at first there should be emphasis of its concrete side with exercises in informal deduction followed by the more formal deductive geometry, omitting many axioms and taking for granted ordinary properties of measurement and

[1] It is taken that Professor Moore does not mean "only those theoretical processes."

motion, early leading up to co-ordinate geometry and *calculus in a concrete way (*calculus is debatable) without taking into account those wonderful related discoveries in abstract geometry which should be reserved for later college and university years. It would be undesirable to attempt to make complete reforms in arithmetic, algebra and geometry all at once.

5. The laboratory method based on a two-period plan preferably, as a means of reform:

(a) This reform calls for a thorough laboratory system or method of instruction, handling pupils individually or in groups so as to develop in each pupil a true spirit of research and appreciation of the practical and theoretical methods of science, the graph and the number and measure relations mutually aiding each other.

(b) Though it may be admitted that it is impossible to concentrate on more than one thing at a time, yet the problem, given as a matter of course, should involve the use of instruments, but should not be given specifically to show their use. Also in teaching one thing relations should be illuminated by the use of others, and the smaller elements of mathematical routine should be attached to and used in laboratory problems.

(c) All results should be checked by at least two distinct methods so as to render the student independent of authority, while in practical problems the amount of error permitted in the final result should be indicated.

(d) The formal proof of a theorem should be preceded by concrete computational or graphical or experimental means so as to awaken appreciation of the importance of the theorem and a desire for formal proof.

(e) Each student studies things and not words, utilizes the experiences of others in close co-operation with the instructor and all are mutually benefited.

(f) The students in the secondary schools will see the vital relation between the fundamental relations of trigonometry, analytic geometry and calculus, when they are closely connected with concrete phenomena and practical education, and with such momentum, in college will be enabled to develop individual and effective interests even in formal abstract mathematics.

(g) Such a full secondary mathematical course in laboratory methods in mathematics and physics (again this unification is a question) is best for all who expect to specialize in college whether in pure mathematics, or pure physics, or in mathematical physics, or in astronomy or in any branch of engineering, but this is to be done gradually, by evolution not revolution, by voluntary close co-operation of teachers and students, by use of computational and graphic methods, colored inks and chalks and by laying emphasis on the comprehension of propositions.

(h) Teaching should be made a profession and the junior colleges by means of the laboratory method can offer training courses in the pedagogy of secondary mathematics in a wonderfully effective way, courses which the college graduates sadly lack, however well trained they may be in content work.

6. Secondary schools are somewhat hampered by specific college requirements and there should be conferences so as to lead to close co-operation.

7. A national organization of science teachers should be formed.

8. The American Mathematical Society should give continuous expert attention to the matter and should form two divisions: (1) Research; (2) Pedagogy.

9. All organizations should take up the matter and form a clearing house for all ideas.

D. Goals Set Up That Have Not Been Realized Or Proposals as to Which There Is Still Controversy

Before stating these, attention is here called to the fact that in Professor Moore's address (and in nearly all of the literature at that time and for some years afterwards and indeed in late years) reference is always made to the boy and not to the girl.

1. That the movement for the enlargement of strong secondary schools by the addition of two years of junior college and the last two or three years of the grades to form an institution like the German gymnasium or the French Lycée will afford a fine opportunity to carry out these reforms is doubtful or at least debatable.

2. That colleges, especially junior colleges should proceed independently to reform their elementary college courses without waiting for the secondary schools to reform their courses and that the secondary schools should likewise proceed independently to reform their courses without reference to the elementary schools is doubtful or at least debatable for many reasons. It is acknowledged that much material will have to be transferred to the lower schools. This with changes in methods would cause an unpleasant hiatus.

3. That the secondary schools should proceed to organize algebra, geometry and physics into a thoroughly coherent four years' course (a suggestion made several times) comparable in strength and closeness of structure with the four years' course in Latin to be given by men who have received expert training in mathematics, physics and engineering is doubtful, or at least debatable. It is to be here noticed that the readjustments in the Latin courses have been very great since Professor Moore delivered his address.

4. That under the laboratory system outlined there should be a thorough laboratory system or method of instruction in unified mathematics and physics, and whether there can be a double period for such laboratory work is doubtful or at least debatable.

5. That the fundamental problem is the unification of pure and applied mathematics by so arranging the curriculum that no branching shall be recognized is doubtful or at least debatable.

6. That there shall be a continuous relation of mathematical problems with problems of physics, chemistry and engineering is doubtful or at least debatable.

7. That the average boy will understand the ideas advanced under the laboratory method which illustrate the uniformity of convergence is doubtful or at least debatable.

The above is an earnest attempt, faulty though it is, to give some idea of Professor Moore's remarkable address delivered nearly a quarter of a century ago. The more one studies it in the light of present day conditions, the more one is impressed with its prophetic character.

The views expressed are not put down in a dogmatic form. In many cases they are opinions based upon the then known

facts and conditions or are in the form of inquiries; in many cases they are expressed as component parts of a vision; in many cases views expressed in one part of the address must be read in connection with views on the same subject advanced under another part of the address to obtain their true significance.

Since then slowly but surely many of the barren spots of mathematics have been removed and the whole subject has been made more vital. The foundations for effective co-operative work were laid in 1902 and 1903; the objectives to be reached, which had long been advocated by so many, though not with the same emphasis have been more or less harmonized, though, of course, all advocates cannot come to the same conclusion.

The World War served to show that common understandings, mutual respect of those holding divergent views, mutual sacrifice for the common good, and kindred attitudes were necessary and desirable not only for war, but for peace activities, not only in trade but in education. The necessary reorganization of conditions necessitated readjustment, readaptation to life's activities and preparation to meet them, while the methods used during the war necessarily had an effect on all educational, social and governmental activities.

For some time the attitude towards all things has been a critical consideration of, and interpretation of ascertained facts to be modified if necessary as the field is extended and the methods of appraising and measuring the values and relations of those facts are perfected.

The tendencies in mathematics as viewed by progressives have been in existence for many years. Those who were more conservative have resisted too great encroachment. Each side had advocates of unquestioned ability and integrity. So long as there was no opportunity for actual meeting and friendly unofficial discussions, there were long range controversies productive of further misunderstandings. This difficulty is being removed and mutual confidence has been established which marks a red letter day in the progress so much desired by all who have in mind the proper development of the child as a social unit of most worth to himself, to the community, to the state.

This mutual confidence which is gradually expanding has made it possible to employ a scientific method of approach in mathematics akin to that used by scientists and especially by the

medical profession: testing by laboratory methods and other methods of analysis for the properties and characteristics of the product under consideration, and then modifying these results by all the other considerations which go to make up the true estimate that should eventually hold.

This is a long and tedious process for the scientist and especially for a member of the medical profession, while the supreme ability, that of accurate diagnostics is even now in its infancy.

The application of the process to education and especially to mathematics necessarily must be slow, and set backs must be expected through lack of understanding of its purpose, its mode of application and action taken on results. An outstanding contribution during recent years has been the introduction and the development of the scientific method of testing in mathematics for: abilities to be developed; useful knowledge to be acquired; proper material to be obtained; sequence of topics in textbooks linked with and based on environment with gradual proper development to prepare for real life:—in short a scientific method of testing so as to accomplish the objectives which have been so earnestly desired and which heretofore have been based on individual opinions of experts which were controlling and rightly so within their special sphere of influence but which might or might not apply in all localities.

The scientific method in mathematics is now entering upon a second and necessary phase—that of the extension of the tests and the perfection of tests already used based upon the records of actual results properly tabulated and extending over a number of years and scientifically checked with actual results.

The makers of these tests are the last to urge that they are sacrosanct. It is claimed, however, that they are of inestimable value in the hands of the experienced teachers as well as of the inexperienced teacher far away from help. These tests do not constitute a universal panacea to cure all mathematical ills nor are they machines to be used by any one to decide the ultimate fate of a pupil. Such work is for the expert. Much less are they to be used indiscriminately for diagnostic purposes. Such work[1] is in the stage of infancy.

[1] These tests, not in use when Professor Moore delivered his address, have confirmed nearly all of his views in a most impressive manner, while at the same time they afford a very sure basis, as pointed out, for future work.

The outstanding results of the past twenty-five years of progress have been pointed out and need not be restated here. The same spirit of mutual help and confidence in impersonal methods scientifically applied will continue and the next few years will witness a wonderful progress in all matters pertaining to things mathematical, in comparison with which the progress of the past twenty-five years, great though it is, will seem very small.

THE FOURTH DIMENSION AND HYPERSPACE

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When I was asked to speak to you twenty minutes on the subject of the Fourth Dimension and Hyperspace, I at first refused, for I thought to talk to mathematicians and scientists about the fourth dimension would be as stale as the news that the earth is round, but after questioning a few of my friends as to what they understood by the fourth dimension and hyperspace, I found that very few of them had a clear idea of what it meant. Not that I know anything about it, for I like to belong to the School of Socrates. When Socrates lived in Athens, it was reported that he was the wisest man in Athens, and Socrates said that he thought they were right, for he said that he knew nothing, and knew that he knew nothing, while the others knew nothing, and thought they knew something. Now I claim to know nothing, and know that I know nothing, but I will attempt to explain to you a few things about what the others think they know.

I will attempt to explain to you the essential character of hyperspace and hypersolids; some of the evidences of a fourth dimension, and a few of the latest developments of the subject; also the utility of speculations on the fourth dimension and hyperspace.

The fourth dimension, as you all know, is a space at right angles to length, breadth, and thickness. Because we are not conscious of fourth dimensional space we must have recourse to analogy to help us understand it. We ascertain the relations between two dimensions and three dimensions and then establish these relations between three dimensions and four dimensions. Just as a plane moving in a direction not contained in itself would generate a three space, so a three space moving in a direction perpendicular to its three dimensions would form a hyperspace. Beginning with a straight line, draw another line perpendicular to it, that will determine the direction of the second dimension. At their point of intersection, draw a third line perpendicular to the first two, that will determine the direction of the third dimension. Now at their intersection, imagine a line drawn perpendicular to those three, that would determine

the direction of the fourth dimension. Obviously, it would contain the point of intersection, but the remainder of the line would stretch out in an unimaginable direction from the inside of that point. The fourth dimension of a solid object lies in a direction within it. A line perpendicular to those four lines would lie in the direction of the fifth dimension and so on.

With the principles of plane and solid geometry, and with making only one assumption, that a line can be drawn perpendicular to three space, we can build up an entire geometry for the fourth dimension. The fourth dimensional geometry is built up by analogy. Just as one reasons what one would do to figures in a plane to form solids, so by analogy one reasons what would be necessary to do to solids to form hypersolids. Just as a square has four corners, and a cube eight, so a hypercube would have sixteen corners. A square is bounded by four lines, and if you move the square through space perpendicular to itself to generate a cube, we find that the cube has the four edges of the square in its first position, the four edges of the square in its final position, and that each corner of the square traced an edge as it moved through space, or twelve edges. By analogy a hypercube formed by a cube moving perpendicular to three-space would have twice the number of edges of the moving cube plus an additional edge for each corner of the cube, or thirty-two edges. The cube is bounded by six planes, or twice the number of planes in the moving square plus a plane for each edge of the square, so the hypercube would have twice the number of planes in the moving cube plus a plane for each edge of the cube, or twenty-four planes. A hypercube would be bounded by eight cubes. One cube at the initial position of the moving cube, one at its final position, and one cube made by each plane of the moving cube as it moved through space. The boundaries of hypersolids would be volumes. The solid of any space becomes the boundary of a corresponding solid in the next higher space. A section cutting a hypersolid into two parts would be three dimensional. A plane cannot separate two parts of a hypersolid any more than a line can separate two parts of a solid. The rules for finding the volume of hypersolids are derived as the corresponding rules for volume are derived in ordinary geometry. Thus the hypervolume of a hypercube would be the volume of the base multiplied by the altitude. Quite an extensive geometry has already been worked

out for the fourth dimension, even including the nature of the hecatonicosahedroids, hypersolids with one hundred faces. These are only a few of the geometrical deductions, and the dramatic results that would accompany a fourth dimension are even more amazing.

By imagining two dimensional beings living in a plane and unable to perceive anything of a third dimension, we get a vivid idea of our own relation to fourth dimensional space, and a consideration of what ought to be their attitude toward any conception of a space of three dimensions, makes clearer what ought to be our attitude toward conceptions of a higher space. Just as in three space a point can go in and out of a square without touching the boundary, so in four space a body could escape from our strongest prison without going through any of the walls. By the fourth dimension you could remove the contents of an egg without breaking the shell. You could take your right glove and flip it over in the fourth dimension and it would come back a left glove. Our densest solids would be open to inspection from the fourth dimension.

Some mathematicians have tried to show that time is the fourth co-ordinate. If time is the fourth dimension, what would one mean by the fifth dimension or the hundredth dimension? This view is too anthropocentric to be plausible. Time is an attribute of all space. Even in a world of one dimension there would be duration or the time it takes an object to move from one position to another. This view arose from the theory that our space is curved in a fourth dimension. We must resort to analogy to understand what this would mean. Imagine a soap bubble with a cone being pushed through it. A being living on the two dimensional surface of the bubble and having no comprehension of a third dimension would see only the successive cross sections of the cone, which he would notice were continually changing, and he would probably call it aging or growth just as we do here. He would have no suspicion that what he is observing is the motion of a solid object in a third dimension. He would see only the two dimensional cross sections of a three dimensional object. Should a four dimensional object come within the field of our vision, we would see only its three dimensional cross section. Einstein claims that our space is curved in a fourth dimension, and that we are being pushed through from a fourth

dimension with a speed equal to that of light, just as the two dimensional surface of a soap bubble is curved in the third dimension and a three dimensional cone can be pushed through it. A three dimensional being could see the whole cone at once, or the whole phenomena occurring which the two dimensional being would interpret as the stream of time. Likewise a four dimensional being could see our whole past, present, and future at once instead of as a stream of time. In any given space, the next higher dimension although spacial has a temporal appearance. Space, time, and motion, are all aspects of relation and some mathematicians claim that they are convertible terms.

The fourth dimension is a perfectly logical concept. The only difficulty comes when we attempt to apply it to our experience and find that it transcends our experience. The hyperdimensionality of space is no longer a question, but the hyperdimensionality of matter is still unproven.

One of the arguments against the fourth dimension is that matter does not move in that direction. Matter may move in the direction of the fourth dimension without our knowing it. Just as the cone being pushed through the bubble would be moving in the direction of the third dimension, but the two dimensional beings living on the surface of the bubble would not know it and would see only the two dimensional cross sections. We cannot assume that because we do not observe a phenomenon it does not exist. We know that there are light waves below the red and above the violet end of the spectrum which are invisible to the eye. There is a possibility that we are a part of four dimensional space with physical limitations which confine us to three dimensional space, and with limitations of our senses which prevent us from perceiving anything outside of this space. Several observations have already been made which seem to indicate that matter also moves in a fourth dimension.

In a plane objects as observed by two dimensional beings would have an infinitesimal thickness in a third dimension, otherwise they would be mere shadows, so in our world objects as we observe them would have an infinitismal thickness in a fourth dimension. It is interesting to note that from the discoveries in recent years which seem to indicate that matter moves also in a fourth dimension, the most evidences of this have been found in the infinitesimal, in cellular and molecular activity.

In our experience we find the phenomena of symmetry only in the minute, such as is produced in plants and animals and in corpuscular and atomic action, and not in the large masses, such as mountains. You all know that symmetrical figures can be made to coincide by giving each a quarter turn toward each other through the next higher dimension. From this point of view, symmetrical figures may be regarded as resulting from a splitting of one figure in a given space and an unfolding in the next lower space. This would explain many of the phenomena of corpuscular action. Also certain chemical isomers appear to differ only in that their molecules are symmetrically instead of identically formed and that in some cases there seems to be free passage from the one form to the other without any manifestation of chemical change. An explanation, though not the only one, would be rotation through the fourth dimension.

Another probable evidence is found in electricity. In a plane the axis of rotation is a point, in three space it is a line, and by analogy in four space rotation would be about a plane as an axis. The electric current forms a closed circuit, which is a geometric image of rotation about a surface. The electric current seems to be a hyper-vortex in the ether revolving about a plane as an axis. The entire nature of electro-magnetic action seems to indicate that the world we know is a cross section of a four dimensional world.

Another interesting possible evidence of the fourth dimension is found in the orbital motion of spheres. Imagine a wire spring suspended in the air and being slowly immersed in a bucket of water. The cross section of the spring where it intersected with the water would be an ellipse which would describe an orbit on the surface of the water each time a coil of the spring was immersed. Now imagine this spring itself being composed of many minor springs of different mass and motion, each intersecting the surface of the water in an ellipse and describing an orbit each time a coil of the spring was immersed. By analogy the cross sections of spiral vortices moving through our ether from a fourth dimension would be spheres moving in orbits, which is exactly what we find in nature. Thus on the hypothesis that the forms of our space are cross sections of four dimensional forms, much of the phenomena of nature would be accounted for, and matter would then be simply vortices in the ether.

From Einstein's discoveries, it seems that our entire universe is a four dimensional hypersphere of finite volume, which by analogy would make it the successive cross sections of one-fifth dimensional helix. For if there is a fourth dimension, there is no reason to stop there. If there is a fourth dimension, it would logically follow that there are any number of dimensions.

Dimension means measurement, and the whole problem of the fourth dimension or any other dimension rests on the nature of space. Space and time have reference to a system of reckoning distances and durations, and one of Einstein's great discoveries is that there are many such systems of reckoning, one as fundamental as the other, and that our frame of space and time is a purely earthly appanage. Einstein's discoveries are the latest development of the subject of space and dimension, and he has arrived at his conclusions by scientific experiments and mathematical calculations. Until recently it was thought that there was an absolute space and an absolute time which could be measured, but we know now that the measurement of space intervals and time intervals are not absolute but relative, and change with the position and motion of the observer. A cubic yard is not an absolute chunk of space. What may measure as a cubic yard to one observer, may measure up as a cubic foot to another in a different position in the universe on a body of different velocity. Time perceptions there would also be different. If our time perceptions were a hundred thousand times faster, events would appear to us a hundred thousand times slower. Anything moving as fast as our express trains would require a delicate experiment to show that it was moving.

Thus it would seem from Einstein's discoveries that our frame of space and time is a purely geocentric view and that space, time, and motion, are not independent entities, but aspects of reason. Einstein calls space and time, "terrestrial adhesions to thought." Kant calls space and time, "synthetic judgments a priori." The majority of philosophers, mathematicians, and physical scientists, agree that space, time, and motion, are not entities, but conceptions of the human mind in its relation to nature. In the last analysis the fourth dimension deals with the fourth step in the reason's comprehension of infinity.

Of what use are speculations on the fourth dimension? Of about as much use as to know whether the earth goes around the

sun or the sun goes around the earth. Space is as properly an object of study as the planets. There is nothing about the fourth dimension that is any more mystical than there is about the other three, and a study of it gives us a clearer idea of the nature of space. It gives us a deeper insight into geometry. Just as a study of solid geometry enlarges our comprehension of plane geometry, so solid geometry is illuminated by a study of hyperspace. It enlarges our mental horizon, and throws light on the nature of our mental equipment. Such speculations sometimes lead to very fruitful results.

USE OF THE INVENTORY TEST IN PLANE GEOMETRY

By LEONARD D. HAERTER
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In the course of the last few years there has been an extensive reorganization of the course of study in mathematics in grades seven, eight, and nine. One phase of this reorganization movement deals with the incorporation of much geometric material. A large amount of simple construction and drawing is being done with the ruler, compasses, and protractor. Through the application of both arithmetic and algebra in the field of geometry the child is becoming familiar with the names and shapes of a large number of plane and solid figures. In addition, many of the simpler notions of geometry are being experimentally obtained by the pupils in these grades.

It was our opinion at the University High School, Minneapolis, Minnesota, that the students beginning the study of plane geometry in the tenth school year must have acquired by means of this revised course of mathematics in grades seven, eight, and nine, many geometric facts which we formerly tried to give them in the first few weeks of the study of plane geometry. We felt that it was most worthwhile to know what information they already had of plane geometry before they began a formal study of that subject.

Accordingly a test was devised covering the beginning of a course in plane geometry. The test, a copy of which appears below, consists of eighty-eight items divided as follows:

A yes-no test of thirty-six items; a completion test of twenty items; a computation test of ten items; and a construction test of twenty-two items.

The following test has been made to determine how much you know about the subject of plane geometry before you actually begin to study it. You are urged to read each question carefully and to answer each one to the best of your ability.

Yes-No

Read the following questions carefully. Write either yes or no before each one.

_____1. Does a triangle have three sides?
_____2. Is the area of a rectangle equal to the base times the height?
_____3. Does a right angle contain 180 degrees?
_____4. Are all right angles equal?
_____5. Are all the angles of a rectangle right angles?
_____6. Does a right triangle contain two right angles?
_____7. Is the area of every parallelogram equal to the product of two adjacent sides?
_____8. Are all the sides of every rectangle equal?
_____9. Is the area of a circle equal to Πd^2?
_____10. Is the circumference of a circle equal to $2\ \Pi r$?
_____11. Is the area of a triangle equal to the product of one-half the base times the altitude?
_____12. Is the sum of the interior angles of a triangle equal to 180 degrees?
_____13. Are two lines in the same plane that do not meet perpendicular?
_____14. If one line is perpendicular to another are the angles formed acute angles?
_____15. Is a quadrilateral a geometric figure of five sides?
_____16. Is an acute angle less than a right angle?
_____17. Can a triangle have two right angles?
_____18. Does a right angle contain 190 degrees?
_____19. Does a straight angle contain 180 degrees?
_____20. Can the sum of two acute angles equal a straight angle?
_____21. Is the sum of two complimentary angles equal to 180 degrees?
_____22. If two angles of a triangle are equal are the sides opposite these angles equal?
_____23. Is the sum of two angles of a triangle ever less than the third side?
_____24. If the sum of two angles is equal to two right angles, are they complementary?
_____25. If the opposite sides of a quadrilateral are parallel, is the figure a parallelogram?
_____26. Are the acute angles of a right triangle complementary?
_____27. Are two triangles always congruent (equal) if three sides of one are equal respectively to three sides of the other?
_____28. Can more than one perpendicular be erected at a given point in a line?
_____29. Are the opposite sides of a quadrilateral always equal?
_____30. Is the area of a rectangle equal to the area of a triangle having the same base and altitude?
_____31. Is an obtuse angle larger than 180 degrees?

_____32. If a triangle has all its sides equal is it equilateral?

_____33. Is the area of a right triangle one-half the product of the legs?

_____34. If two angles of one triangle are equal respectively to two angles of another triangle, is the third angle of the first equal to the third angle of the second?

_____35. If two sides of a triangle are equal, are the angles opposite equal?

_____36. If three angles of one triangle are equal to three angles of another triangle, are the triangles always congruent (equal)?

Completion Exercises

Write one word in each blank to make the sentence state a geometric fact.

1. A line passing through the center of a circle and having its ends in the circumference is called a_____.
2. An angle containing 90 degrees is called a_____angle.
3. In a triangle the line drawn from the vertex perpendicular to the opposite side is called an_____.
4. A rectangle contains_____right angles.
5. If one line is perpendicular to another, the angles formed are_____angles.
6. A line drawn from the center of a circle to the circumference is called a_____.
7. If a rectangle has all its sides equal it is called a_____
8. If all the sides of a triangle are equal, the triangle is called_____.
9. If a triangle contains a right angle, the triangle is called a_____ triangle.
10. In a right triangle the side opposite the right angle is called the _____.
11. Two angles whose sum is equal to 180 degrees are called_____.
12. The_____of a_____is equal to Πr^2.
13. If two sides of a triangle are equal the triangle is called_____.
14. A straight line that touches a circle in only one point is called a _____.
15. A hexagon is a geometric figure of_____sides.
16. The sum of the interior angles of a triangle is equal to_____ degrees.
17. If two triangles have_____sides of one equal respectively to _____sides of the other, the triangles are_____.
18. The line joining the opposite vertices of a quadrilateral is called a _____.
19. The square of the_____of a right triangle is equal to the_____of the_____of the other two sides.
20. An equilateral triangle has each angle equal to_____degrees.

Computation Exercises

1. A rectangle has a base of 20 feet and an altitude of 8 feet. What is the area? Ans._____sq. ft.
2. A triangle has a base of 12 feet and an altitude of 10 feet. What is its area? Ans._____sq. ft.

3. The sum of two angles of a triangle is equal to 120 degrees. How large is the third angle? Ans._____
4. One of the acute angles of a right triangle is 50 degrees. How large is the other acute angle? Ans._____
5. Two parallel lines are cut by a transversal so that one of the angles is equal to 30 degrees. How large is its alternate interior angle? Ans._____
6. An acute angle contains 70 degrees. How many degrees are there in its complement? Ans._____
7. The vertex angle of an isosceles triangle contains 20 degrees. How many degrees in each of the base angles of this triangle? Ans._____
8. A central angle of a circle contains 40 degrees. How many degrees in its intercepted arc? Ans._____
9. One leg of a right triangle is 6 feet and the other is 8 feet. How long is the hypotenuse? Ans._____
10. How many degrees are there in the sum of the angles of a quadrilateral? Ans._____

Construction Exercises

Draw freehand as neatly and as large as possible the figures and lines asked for.

I. The triangle:

1. Draw an isosceles triangle here.
2. Mark the base AB.
3. Draw the altitude to that base.
4. Draw a bisector of one of the angles at the base.
5. Draw a line through the vertex of the triangle parallel to the base.
6. Draw a right triangle here.

II. The quadrilateral:

7. Draw a rectangle here.
8. Draw a diagonal of this rectangle.
9. Draw a square here.
10. Draw a trapezoid here.

III. The circle:

11. Draw a circle here.
12. In this circle draw a radius.
13. In this circle draw a chord.
14. Draw a line tangent to this circle.
15. Draw a central angle of this circle.

IV. Angles.

15. Draw a central angle of this circle.
16. Draw an acute angle here.
17. Mark the angle so that it can be read angle ABC.
18. Draw its complement. Place a (1) in this angle.
19. Draw its supplement. Place a (2) in this angle.

V. Parallels:

20. Draw two parallel lines here.
21. Draw a transversal to these parallels.
22. Mark a pair of alternate interior angles by (1) and (2).

On the opening day of school this test was given to 74 pupils who were beginning the study of plane geometry for the first time. All the pupils had pursued a revised course of study in grades seven and eight, and a course in general mathematics in grade nine. It is interesting to note in the abulations which follow, just how much geometry they knew.

Yes-No Test

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Times missed	0	5	6	10	7	9	34	5	31	28	7	2	23	15	17	5	8	1

No.	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Times missed	1	11	33	20	12	32	23	28	16	6	21	5	18	8	30	7	17	47

Completion Test

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Times missed	22	2	41	6	8	17	12	18	3	27	43	22	38	60	31	3	49	52	51	20

Computation Test

No.	1	2	3	4	5	6	7	8	9	10
Times missed	6	6	8	11	35	34	26	68	36	20

Construction Test

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Times missed	17	14	22	19	45	17	5	21	12	40	9	13	60	63	56	14	38	45	45	11	13	30

Each item of this test was valued at one point and the score of each pupil was obtained by adding together the total number of correct replies that appeared on the paper.

The median score of the class on the test was 63. A brief comment on the summary will be illuminating.

Yes-No Test

1. Seventeen questions were each missed ten times or less.

2. Twenty-five questions were missed less than twenty times each.

3. Only eight questions, numbers 7, 9, 10, 21, 24, 26, 33, and 36, were missed by more than one-third of the class. An examination of these questions will readily explain the frequency of error. Numbers 9 and 10 would be missed almost as frequently by a class who had already studied plane geometry. Number 7

is a trick question, while numbers 21, 24, and 26 deal with the notion of complementary angles, a notion which is readily confused with that of supplementary angles by the pupils.

4. Only one question, number 36, was missed by more than half the class. We have evidence from the results of the Minneapolis Geometry Test, devised by Miss Anna Thomas of Minneapolis, that this question is almost as frequently missed by pupils who have studied plane geometry for one-half year.

Completion Test

1. Seven questions, numbers 3, 11, 13, 14, 17, 18, and 19, were missed by over half the class. This portion of the test was apparently the most difficult.

2. Nine questions were missed twenty times or less. The wording of these questions undoubtedly gave many pupils trouble; while the strangeness of this type of test was also confusing.

Computation Test

1. Only question 8 was missed by over half the class.

2. Questions 5, 6, 7, 8, and 9 were missed by over one-third of the class.

3. Numbers 1, 2, 3, and 4 were missed by less than one-seventh of the pupils.

Construction Test

1. In the construction test, numbers 5, 10, 13, 14, 15, 17, 18, 19 and 22 offered special difficulty. They deal, however, primarily with things met only after many weeks in plane geometry, and with the matter of complementary and supplementary angles.

2. Half the questions were missed by less than twenty persons in the class.

On the showing made by the pupils on the test we decided to spend no time whatever in reviewing these facts as a part of the regular class work. We believed that undoubtedly a brief review by the pupil out of class would readily recall these things to his mind. Each student was therefore given a slip of paper bearing his name and the numbers of the problems missed, and was told to find the proper answers to the questions missed. References were given and a sufficient number of tests posted on bulle-

tin boards for the pupils to consult. After one week the test was given again. The median of the class was raised from 63 to 80. Only one pupil made a score below 50.

A summary of the results made on the second giving of the test follows.

Yes-No Test

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Times missed	0	4	3	1	1	3	20	8	15	17	6	3	14	2	9	2	4	0

No.	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Times missed	0	3	17	0	4	15	12	14	6	1	12	1	7	2	17	8	1	39

Completion Test

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Times missed	14	3	36	2	3	5	8	1	4	8	13	11	10	22	12	2	18	30	43	4

Computation Test

No.	1	2	3	4	5	6	7	8	9	10
Times missed	0	4	2	6	16	9	8	52	13	8

Construction Test

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Times missed	0	0	6	4	26	4	2	2	3	13	2	9	27	22	18	5	25	16	17	3	3	4

Criticisms of two kinds may be raised.

1. University High School has a very select group of students.

In answer to this I would say that this same test was given at the beginning of the school term to 270 pupils at West High School, Minneapolis. This school I believe to represent a normal situation. The pupils here had the same preliminary training as those at the University High School. The median score made by the West High School pupils on the test was 56. I do not know what procedure was followed in this school after the tests had been given, but undoubtedly a very similar improvement would have taken place if a procedure similar to the one used at University High School had been used.

2. Few school systems have this revised course of study which was spoken of at the beginning of this article.

In answer to this I would say that a close examination of recent Junior High School texts for seventh and eighth grades will reveal the fact that a large part of them is given over to simple drawings and constructions which bring into use the names of

geometric lines and figures, and acquaintanceship with geometric facts and terms as contained in the above test. In the ninth grade there appears to be developing a well organized course incorporating many more of the same type of facts through the formula and its applications, and through the notion of similarity and proportion. Unquestionably what is happening is that a consecutive and well organized course of general mathematics is being developed for these grades, of which the geometric phase is but one of the many features.

I believe, due to the progress that this revised course of study, outlined above, is making throughout the country, and due to the results of the test already referred to, that the following conclusions are warranted:

1. Pupils approach the study of plane geometry today with a large store of geometric information. Two or three weeks need no longer be spent in the teaching of names and terms. We may justly begin farther along in our work. Recent trends in reorganization would seem to indicate that this store of facts will be increased from year to year rather than decreased. Moreover, this information which the student possesses has been a gradual acquisition and is an integral part of his knowledge in a way that it never was when taught at the beginning of grade ten.

2. We should now be able to do considerably more work in the tenth year than was formerly possible. Our brighter pupils should very well be able to cover a course as outlined in the National Committee Report in less than one year. One of the following developments seems a logical consequence:

a. A combined course in plane and solid geometry for our brighter pupils in grade ten.

b. A course in plane geometry supplemented by considerable mensuration work in solid geometry.

c. A course in plane geometry supplemented by a considerable unit of trigonometry.

Some such revision will have to take place in grade ten or else I believe that we are not taking full advantage of the reorganization already effected and that is continually being effected in the grades immediately below the tenth.

MEETING THE ATTACKS ON ALGEBRA

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Spring Meeting, May 2, 1925 Association of Teachers of Mathematics in New England

No subject in the high school curriculum, according to some educators, has yielded such unsatisfactory results as Algebra. It has been the subject of most severe criticisms on all sides, and has been held responsible for a great deal of freshman mortality. A leading educator of Massachusetts, in one of his public speeches, has expressed the desire that less time should be devoted to the study of algebra in the high school. An influential body of educators has even gone so far as to ask some of our Massachusetts colleges to reduce their entrance requirements in algebra. We not only find educators loudly declaiming algebra; but this spirit of criticism has carried still farther. We know that many school children hate algebra because their parents hated it, or because other pupils have told them how uninteresting and difficult it is. Parents dread the time when their children must study algebra. In a magazine article of recent date, a father was discussing the education of his daughter. In the course of the discussion he said that his daughter did not go to college because of her intense dislike of algebra.

What has been the result of all this? Algebra, except in the strictly college preparatory schools, has become more and more an optional subject. The majority of pupils now finish their high school course with no broader insight into mathematics than that obtained in their first eight years of school.

Perhaps some of these criticisms are justified, after all. We must admit that there has been general dissatisfaction with the results of teaching of algebra. It is claimed that algebra, as it has been taught, is too remote from life to interest the pupil and has outlived its usefulness as a subject of secondary school instruction. Modern psychologists claim that there is no such thing as mental discipline; therefore mathematics teaching as a whole has no value. I think that, although we are unwilling to do so, we must admit that traditional algebra has little educational value. If algebra consists chiefly in the acquisition of certain facts, in the manipulation of symbols, and in the solution

of a few book problems, it deserves to be the victim of propaganda. It must fight a losing fight and should be "pitched overboard," for there are no tangible results forthcoming from the algebra as taught in the old days.

It is our job as mathematics teachers to dispel this current idea that algebra is as useless as it is difficult. We must endeavor to define and to teach algebra so that it becomes interesting and educationally valuable to young people. We must present it in such a way that an average pupil can grasp it and reason it out as he goes along. Pupils must be taught to handle thoughts and not merely symbols of thought. We should forget the mechanical notions and see algebra through the eyes of the pupils, and relate the subject to them.

We can do a great deal toward dispelling the doubts as to the usefulness of algebra by the proper presentation of the subject matter. Organize what the pupil knows before introducing new facts. Each fact and process should be thoroughly mastered by the pupil before he is plunged into a new difficulty. Make Algebra real and interesting to the pupil right from the outset.

The real impetus to this movement of reform in algebra was given by the report of the National Committee on Mathematical Requirements in 1916. Since that time a wave of reform has been gradually gaining headway in mathematics, and the new algebra is steadily making progress in the high school curriculum. Many new textbooks have appeared recently, carrying out the recommendations laid down by this committee. These books reflect to a very high degree the present trend of thought. It seemed to me that it might prove interesting to make some comparisons between the presentation of topics in the traditional algebra and in the new algebra, and thus see what is being done to meet the attacks that have been constantly made on algebra.

The topics of Algebra as most of us were taught in high school were presented in some such order as the following: A short introduction explaining the use of symbols in algebra with a few equations to illustrate the use of symbols, explanation of the terms, factor, exponents, coefficients, parentheses, radical sign, etc.; a short exercise, as a rule, in evaluating algebraic expressions, in which the pupil was taught the correct order of fundamental arithmetical operations.

With this body of definitions and explanations as a background, the pupil was plunged immediately into the chapter called "positive and negative numbers." Then the mechanical rules for addition and subtraction of monomials were given with numerous examples. Following this the law of signs for multiplication and division was given. To be sure, some of these rules were explained, e.g.

$$-4 \text{ times } 3 \text{ means } (-4) + (-4) + (-4) = -12.$$

I wonder how many pupils really understand this explanation, whereas — 4 times — 3 means that — 4 is to be subtracted 3 times, etc. Then addition of monomials and of polynomials was presented next. There followed a chapter on simple equations, which at first glance seemed to have no connection with the preceding material, although we later see that some of the equations involved the uniting of similar terms. Next came subtraction of polynomials, more equations, parentheses, multiplication of polynomials, more equations, but this time perhaps involving parentheses, division, more equations, etc. Each new topic was presented, but the need for it was not apparent. The pupil must learn a great many rules, then he is given problems and exercises in which to apply those rules. I do not mean to imply that the pupil should learn no rules in algebra; but the point we should make is that the pupil should first see the need for a process; this process is rationalized as far as is permissible in the reasoning power of the pupil. He understands the process and is ready for the rule. Certain processes gradually do become mechanical, but this mechanical handling of them is justified provided the pupil thoroughly understands what he is doing.

To go a little farther with the traditional algebra. In the first year work the pupil was expected to cover many cases of factoring, including the sum and the difference of cubes. The finding of the highest common factor by the Euclidean method was enough to discourage any pupil, especially when he could see no immediate use for it, for the fractions he dealt with should not be difficult enough to require this method of finding the highest common factor. I might continue thus and show how much was expected of the pupil, but I know this would only prove very tiresome.

As a contrast to this, let us examine part of a table of contents of one of the recent algebras.

I. Formulas—Equations.
II. Formulas of Mensuration—Approximate Computation.
III. Application of Formulas to Science and Industry.
IV. Linear Equations—Graphs.
V. Positive and Negative Numbers, etc.

Note the late introduction of positive and negative numbers. Let me quote just a little from the preface to indicate just what steps are taken to improve the teaching of algebra and to overcome so many of the objections to traditional algebra.

"The approach to algebraic symbolism and technique is through well-known formulas. Continued emphasis is placed upon rational methods of calculation and the determination of the reliability of numerical results when computed from data obtained by measurement. The simple and relatively late introduction of the negative number and the definite attempts to overcome the difficulty of handling of the signed number are some of the features of the books," etc.[1]

The "New Algebra" is built chiefly around the formula and the equation. The pupil is introduced to algebra through the formula, with which he is already somewhat familiar. The algebra is reorganized from the problem-solving point of view. The very first equations should be those that solve problems, for we certainly can not teach equations until we know how equations are obtained. Every new difficulty should be introduced by a problem, so that the pupil can discover for himself that he needs the new process to solve that particular problem.

Rather than go into detail concerning all the topics in the "New Algebra," I shall discuss the treatment of two which are presented in a manner very much different from that found in the traditional algebra. I refer to the solution of equations, and to the handling of problems.

The "Great Equation Law" as one author calls it is constantly emphasized throughout: "What ever we do to one side of an equation, we must also do to the other." The various types of equations are presented in the order of their difficulty; *e.g.*

I. $4x = 12$.
II. $3x - x = 16$.

[1] The above is taken from the Revised Edition of Junior High School Math. Third Course—by Vosburgh-Gentleman-Hassler.

Then $3x + 5 = 17$ in which we should like to find $3x$ first. But we have 5 more than $5x$. Therefore we subtract 5 from both sides of the equation, or $3x = 12$.

Perhaps some may ask how we can handle $2y - 25 = 43$ when we have not taken up negative numbers, or addition or subtraction. We should like to find $2y$. But we are short 25 of $2y$, i.e., we have a shortage of 25. To make up this shortage we add 25 to both sides of the equation, $2y = 68$. The minus sign indicates subtraction, or a shortage and that is all there is to it. The word transpose is never used. We teach the pupils a common-sense method of operation upon equations and not some mechanical process. It is important to note that the pupil will never have the occasion to divide by a negative number in solving these simple equations, because he makes up shortages if there are any. The question may come up: how shall we handle this: $2x = -6$. If $2x$ is a shortage of 6, then $1x$ or x is one-half of that shortage or -3. In other words, we treat quantities with minus signs before them as shortages. Thus we carry on our development of the equation dealing with those involving parentheses, and those involving fractions which have monomial denominators. We always introduce each new type by means of a problem and then give a common-sense way of arriving at an answer. No work should be accepted that is not carefully checked. To be sure, the old algebra encouraged checking. We do more than that: we insist on checking and accept no work as complete until it has been carefully checked. There are two more points I wish to indicate in connection with equations: A series of steps have been built up to aid in the solution of equations. The form is due to Mr. Evans of Charlestown.

In my own work with ninth grade classes, I have found these exceedingly helpful. Most of you are, no doubt, familiar with these:

(1) Perform any indicated multiplications.
(2) Unite similar terms in each member.
(3) If there are any terms with minus signs before them, add enough to make up these shortages.
(4) Subtract from each member the smaller unknown term.
(5) Subtract from each member any known term that stands beside an unknown term.
(6) Divide both members of the equation by the coefficient of the unknown.

The second point is the use of a convenient symbolism to enable the pupil to describe what he has done. This also, I believe, is due to Mr. Evans, and is an outstanding feature of the new Algebra. The pupil decides what he must do before he actually does it.

$$\text{①}\quad \frac{2x}{3} - \frac{2+x}{7}\,\frac{1}{} = x - 2$$

②	$14x - 3\ (2x + 1) = 21x - 42$	① × 21
③	$14x - 6x - 3 = 21x - 42$	② same values
④	$8x - 3 = 21x - 42$	③ Same values.
⑤	$8x + 39 = 21x$	④ + 42
⑥	$39 = 13\ x$	⑤ − 8x
⑦	$3 = \quad x$	⑥ ÷ 13

Check

$$\frac{2x}{3} - \frac{2x+1}{7} = x - 2$$

$$2 - 1 = 3 - 2.$$

Ans. $x = 3$.

The discussion of the equation is very sketchy, indeed; but I hope I have succeeded in showing how different its development is from that in the traditional algebra.

The study of mathematics should result in the development of power, rather than in the acquisition of facts and ability to handle difficult and intricate manipulations. There is no better way of developing that power than by the solution of problems. One of the common remarks we often hear from pupils is: "I like Algebra, but hate problems," as if problems were separate and distinct from algebra. We find model examples in most of the algebras solving problems, but the pupil is not told how to go about it in such a manner that he will be able to handle any problem that may come up in the course of his study. In "Everyday Algebra," a book to be published shortly by Houghton Mifflin Co., the author has used problems as the vehicle of introducing each new algebraic fact. He has set up certain steps in problem solving which are well worth considering at this point, for they again show what a decided step in advance the new algebra has taken to overcome the great dislike of problems.

I. The first step is algebraic analysis, i. e. decide what quantities are to be listed.

II. The second step is to express related quantities algebraically. Consider (1) which quantity will be represented by a single letter (2) how the other quantities are related to the quantity you have selected and (3) the total, which can be represented both algebraically and numerically.

III. The third step is to form an equation.

IV. Solve the equation.

V. Test the answer to see if it meets the condition of the problem.

He points out that there are three possibilities in setting up the equation:

(1) We have 2 abbreviations for the same quantity.

(2) We use some fact stated or implied in the problem, but not used in the list.

(3) We substitute in a formula.

There are various other topics in the new algebra, that are well worth considering, such as the late introduction of negative numbers and of algebraic addition and subtraction, approximate computation, extensive use of various kinds of graphs, etc., all of which are treated in a manner that indicates very clearly that mathematicians have taken up the cudgels in defense of algebra.

Some teachers may say: "Well those ideas are all very fine; but can we follow them out with pupils who are preparing for college? Have we the time to do it?" Our answer is, "Why not?" Shouldn't the pupils who are going to college also be trained to use their common sense and understanding in algebra? The College Board has indicated that it realizes and appreciates the present tendency toward improvements in the teaching of algebra by its willingness to reduce in its requirements a great deal of algebra dealing with algebraic technique and to omit certain topics previously required, such as the square root of a polynomial, the finding of the highest common factor as a separate topic, etc.

We, teachers of mathematics, are fortunate to be a part of the mathematical world at a time when such a tremendous reorganization is taking place. We have seen Algebra attacked on all sides with fatal results to its position in the high school curriculum. We now see the advent of the new algebra which bids fair to meet all these attacks and to restore algebra to the high position it deserves in education.

THE SOLUTION OF QUADRATIC AND CUBIC EQUATIONS ON THE SLIDE RULE

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Any quadratic equation may be expressed in the form

$$x^2 + bx + c = 0. \quad \text{.....(1).}$$

If the roots are real it is only necessary to find two real numbers which product is c and whose algebraic sum is $-b$. Hence to solve a quadratic equation of this type on the slide rule set the cursor at c on the A scale and move the slide until the sum of the numbers under the cross hair on the B scale and over the index on the A scale is equal to $-b$ these two numbers are the roots.

Example: (1) $x^2 + 5x + 6 = 0.$ .....(2).

Set the cursor at 6 on the A scale. Move slide to right until 3 on the B scale is under the cross hair: then the index is under 2 on the A scale. Therefore 3 and 2 are roots.

Example: (2) $x^2 - 3x - 10 = 0.$

Set the cursor at 3 on the A scale, move slide until 5 on the B scale is under the cross hair and the index is below 2 on the A scale. Then $+5$ and -2 are the roots.

Even if the roots are imaginary, they may still be found on the slide rule by means of the following lemma. The condition for imaginary roots in (1) is that $\frac{b^2}{4} - c < 0$. If now c' be made equal to $\left(\frac{b^2}{2} - c\right)$ then $\frac{b^2}{4} - c' = \frac{b^2}{4} - \left(\frac{b^2}{2} - c\right) =$

$$c - \frac{b^2}{4} > 0.$$

Therefore to transform a quadratic equation with imaginary roots into the corresponding one with real roots, multiply b by $\frac{b}{2}$ and subtract c. Having found the real value subtract $\frac{b}{2}$ and multiply by c. This will give the imaginary part of the complex root, the real part being $\frac{b}{2}$.

Example: Solve $x^2 - 6x + 15 = 0$. .....(3)

$$b \times \frac{b}{2} = \frac{36}{2} = 18, \quad 18 - 15 = 3.$$

Therefore the corresponding equation with real roots is

$$x^2 - 6x + 3 = 0. \quad \text{.....(4)}$$

The solution of (4) on the slide rule is 5.45 and .55. Subtraction of $\frac{b}{2} = 3$ gives 2.45.

Therefore the roots of equation (3) are

$$3 \pm 2.45\, i \quad \text{.....(5).}$$

Runge[1] has shown how to use the slide rule with inverted slide to solve cubic equations of the form

$$U^3 + 5U = 3. \quad \text{.....(5).}$$

But this is also solvable as follows without inversion of the slide. If we read u upon the C scale then u^2 is just above it on the B scale. Then the product $u(u^2 - 5) = 3$. Hence set the cross hair of the cursor at 3 on the D scale, move the slide to the right until the number u^2 on the B scale minus 5 is equal to the number on the D scale below the index of the slide. (Fig. 1.)

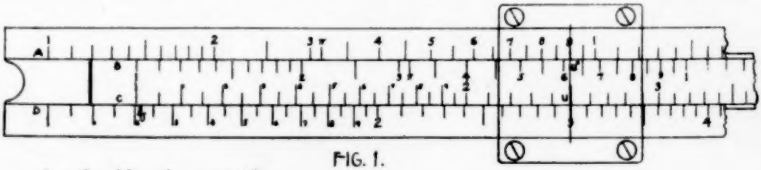

FIG. 1.

Again if u is negative

$$u^2 + \frac{3}{u} = 5$$

Hence move the slide to the right until the number under the cross hair on the B scale plus the number below the index of the slide on the D scale equals 5. Read $u = 1.84$. (Fig. 2.)

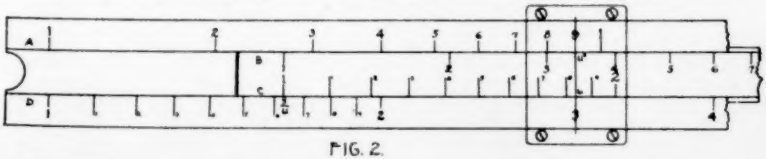

FIG. 2.

[1] Runge: Graphical Methods; p. 46.

There must also be another negative root. To find this move the slide to the left until as before $u^2 + \frac{3}{u} = 5$. Then $u = -.66$.

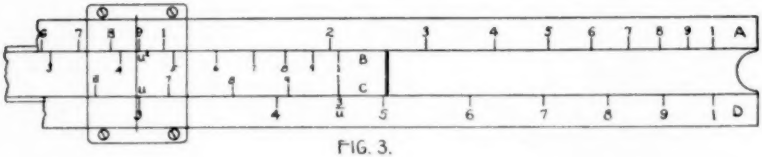

FIG. 3.

If the cubic is in the form $u^3 + au^2 = b$, it may be readily solved for one real value. Let us solve $x^3 + 3x^2 = 20$ or $x^2(x + 3) = 20$. Set the cursor to 20 on the A scale. Move the slide to the right until the number on the B scale under the cross hair minus 3 is equal to the number on the D scale below the index of the slide is the solution 2. (Fig. 4.)

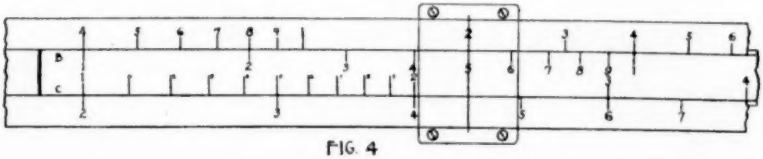

FIG. 4

Another interesting solution of such an equation

$$x^3 + 3x^2 = 54 \text{ or } x + 3 = \frac{54}{x^2}$$

is: set the cursor at 54 on the A scale, then read x^2 on the B scale and x just below it on the C scale. The index of the slide will always point to $\frac{54}{x^2}$ on the A scale. Hence move the slide to the right until the number under the cross hair on the D scale plus the three of $(x + 3)$ is equal to the number $\frac{54}{x^2}$ on the A scale.

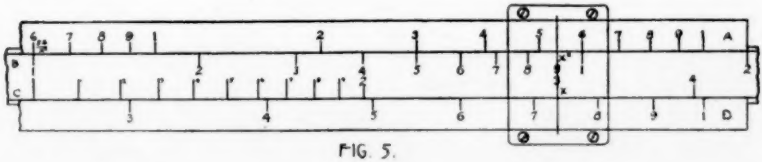

FIG. 5.

The following example shows a general method of solution in which all three roots of the cubic are found. If the cubic is in the form $x^3 - 5x - 3 = 0$ and if a, b, and c are the roots, $a + b + c$ must equal zero, since x^2 is missing: also $abc = 3$. Therefore, if the first root is found as in (Fig. 6)

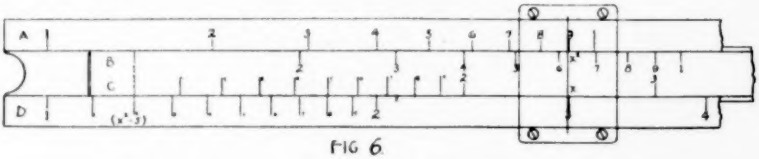

FIG 6

$x = 2.5 = a$. Then $b + c = -a = -2.5$

$$\text{and } bc = \frac{3}{a} = \frac{3}{2.5} = 1.2$$

Accordingly having found the first root, move the cursor over to 1.2. Then move the slide to the left until the number under the cursor on the C scale plus the number below the index on the D scale will together equal -2.5. (Fig. 7.)

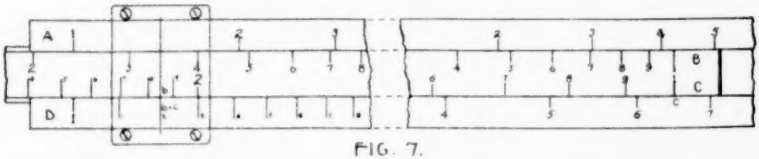

FIG. 7.

Complete solution $a = 2.5$, $b = -1.84$, $c = -.65$.

SOME INTERESTING SIDE LIGHTS UPON ELEMENTARY MATHEMATICS, INTRODUCING A NEW EXPANSION OF BINOMIAL THEOREM

BY MRS. L. H. MEEKS
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In this article I hope to call attention to certain diversions in the study of elementary mathematics as pursued in high schools today.

The value of any diversion in the study of a given subject is too well known to merit any particular comment here. For instance the craze in cross-word puzzles, which has recently swept over the country, has exerted a noticeable influence in word study, and although this stimulant may be short lived, its influence will undoubtedly be felt for some time in the increased use of the dictionary. While cross-word puzzles may have little or no bearing upon mathematics it serves, however, to illustrate the power of such diversion, in creating or reviving an interest in any particular thing.

We have a great many interesting and fascinating combinations in the study of mathematics and if such things are pointed out to the student at the proper time it may relieve a great deal of the humdrum during the study of this subject. It has been in following some of these combinations that I have stumbled upon some interesting facts.

In this article I wish to show some relations that exist between powers of numbers. Let us first look at the relations that exist between the squares of two numbers. I early learned that the difference between the square of two numbers is equal to the product of their sum by their difference. Let us take two numbers—say 5 and 8 for example.

Sum of the two numbers	equals 13
Difference between the two numbers	equals 3
Product of the sum and difference	equals 39
Now the square of 8	equals 64
Now the square of 5	equals 25
Difference between the squares	equals 39

It should prove refreshing to the student to try out many other pairs of numbers.

Now let us consider the cubes of the numbers. I have discovered that the difference between the cubes of two numbers is the product of the difference between the two numbers and the remainder obtained from deducting the product of the two numbers from the square of their sums. An illustration may make this clearer. Taking for example the same two numbers 5 and 8—

Sum of the two numbers equals 13
Square of the sum of the two numbers euals $13 \times 13 = 169$
Difference between the two numbers equals 3
Product of the two numbers $5 \times 8 = 40$.
Square of the sums minus the product $169 - 40 = 129$
$129 \times$ difference $= 129 \times 3 = 387$, the difference between the cubes of the two numbers.
$8^3 = 512$
$5^3 = 125$
Dif. $= 387$

This should also be tried out on several other combinations.

Now that these experiments have been made with the ratios that exist between the squares and cubes of numbers of one term let us try out the binomial.

The process used in elementary algebra is rather tedious and the well-known formula for the expansion of a binomial is also tedious, so I have used another system of expansion which can be made up in tabular form and may prove to be refreshing to the student if introduced at the proper time. The table which follows and is labeled Fig. 1 is made out for a binomial expanded to the 10th power. The student should continue this still further. In the table the numbers indicate the coefficient and it is understood that the exponent is one.

The method of constructing the table is quite simple, beginning at the top we start with a simple integer, on the line below we add one integer making the term a binomial of the first power—the next line or second power is obtained by starting at the left with one, the next term is obtained by adding together the two terms immediately above, etc., always adding one at the extreme right, each succeeding line being obtained in a like manner.

	1
1	1 + 1
2	1 + 2 + 1
3	1 + 3 + 3 + 1
4	1 + 4 + 6 + 4 + 1
5	1 + 5 + 10 + 10 + 5 + 1
6	1 + 6 + 15 + 20 + 15 + 6 + 1
7	1 + 7 + 21 + 35 + 35 + 21 + 7 + 1
8	1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
9	1 + 9 + 36 + 84 + 126 + 126 + 84 + 36 + 9 + 1
10	1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 + 1

Fig. 1.

Formula. $(a + b)^n$

$$a^n + na^{n-1}.b + \frac{n(n-1)}{1.2} a^{n-2}.b^2 + \frac{n(n-1)(n-2)}{1.2.3} a^{n-3} b^3 + \ldots + b^n$$

The use of the table is quite simple also after it has once been constructed, but it must be borne in mind when writing a power, that the first term must be arranged with the exponents in the descending powers and the second term with the exponents in the ascending powers. As an illustration let us find the sixth power of $(a + b)$ looking at the table and finding the form for the sixth power we proceed to write

$$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6.$$

Now is that not easier than applying the formula or multiplying it out in five tedious operations?

In looking at the table one may find a great many interesting combinations, for instance it is noted that the figure takes the form of a triangle, a very stable form, that the two outside lines are ones, each of the second lines advance in arithmetical progression, i.e., they are consecutive numbers, that all the lines are advanced by adding to the last number on a given line the number above and ahead in the former line. In looking at Fig. 1 we see the fine dotted lines connecting numbers together, for instance in the second row on the left side we see $1 + 1 = 2$, in the third row $3 + 3 = 6$, and so on for each line throughout. A little observation will reveal a great many of these coincidents.

WHY IS IT?

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West High School, Cleveland, Ohio

I have just looked through copies of 10 current high school geometries and in not one of them can I find a direct and explicit proof of the fact that if the square on one side of a triangle equals the sum of the squares on the other 2 sides, it is a right triangle. In some books it is proved indirectly by proving its converse and its opposite, but no attention is called to the fact. Some authors use the fact in problems without having proved it or mentioned it and these same authors seem worried in their opening chapters if the converse of any little proposition is assumed to be true without giving the proof of it.

I suppose there are as many proofs of this fact as there are of its famous converse, but they turn out in practice to be somewhat trickier than their relatives. The easiest and most satisfactory one that I found is the converse of the first proof usually given. Let h^2 equal a^2 and b^2. Take a length c on h adjacent to a so that ch equals a. Call the rest of hd and then dh equals b^2. Hence c/a as a/h which proves two triangles similar as they have the included angle in common. Same with dh equal b^2. By the equal angles in the similar triangles one angle of the triangle can be proved to equal the other two.

9B ALGEBRA

The teaching of beginning algebra can be a real mental stimulus if the student is led into it through the door of experiment and discovery rather than driven in in close formation with the 'goose step.' Too many rules and too many exercises of problems of exactly one kind enables and encourages students to get by with a minimum of analytic and creative thinking, largely by mere remembering. Just another set of facts that are so because it says so in the book and the teacher says so. Students that are compelled to get algebra this way or not at all should not be taking it. They will not be able to use it or go on with its study. It is the presence of so many of these students that makes so many teachers feel compelled to drill it into them. It would be better for them if a real effort were made to get them to think.

As it is they are doped with rules and passed on to geometry or passed out of school. As for those students of sufficient mental caliber to appreciate for themselves the why and wherefore of algebra, it is not a square deal to them to be deprived of the opportunity to do so.

After years of trying to get algebra across in the order and manner presented in the algebras supplied, this suggestion gradually evolved. Present to the students only word problems and only take up such technical algebra as is necessary in the solution of those problems. Thus the immediate problem would supply the incentive for the study of the necessary algebra and give satisfaction in its use and the immediate relation between x and the value in the problem that it represented furnishes a less slippery handle to get hold of the difficulties of using letters to represent numbers. The student will see just where the difficulties arise that are not present in arithmetic. The necessity of quantities less than 0 is met right where it lives and can be studied in its natural habitat. The parenthesis comes in as a needed form of expression and not as another aggravation.

Back of this is the assumption that even in the algebra class the future citizen is the thing and not the little bit of algebra. How small this bit is, is appreciated by those who ask their students to use some of their year of algebra in the geometry or by the teacher of advanced algebra. Algebra presented in the way indicated would give a laboratory subject the equal of physics or chemistry in opportunity to experiment and draw and generalize conclusions. The rules of algebra such as 'like signs give plus' will have to be introduced as their use is a mental economy that cannot be ignored, but they should not be introduced until there is a background of personal experience which makes the rule more than mere words.

This experiment could not be tried out in its pure form in a regular school, but I have tried a modification of it. For over two months, using a current algebra, we ignored everything in the book except the word problems. The students worked at their seats in class and asked help by raising their hands. Anything of general interest was talked over at the board. At first the formation of the vocabulary and the translation into an algebraic equation got all the attention, but soon the problems of technical algebra appeared. The difficulties of the translation

were of course never completely conquered, but I see no reason for stepping around them on that account. Algebra will be useless to one who has not sufficient mental power to form the equation from the problem and the battle might as well be fought here as anywhere. There is strength to be gained in fighting and character to be gained in facing the issue.

The temptation to use two unknowns which suggested itself to the students quite naturally was resisted for a while, but in time we took up simultaneous equations and though I only showed them elimination of one letter by addition or subtraction, one boy discovered elimination by substitution and a boy and a girl eliminated two of three letters without any help. We then graphed equations for two weeks. From this we went to multiplication, oral and then long, then direct to long division, and if properly treated this furnishes one of the best exercises in algebra. The division is put in the same form as the multiplication, the divisor at the top and the dividend in place of the product and the suggestion is given that we are to find out what the divisor was multiplied by to give that product. With no other help some of the students fill in all the terms for the hardest problems in the exercise. Others must be reminded how columns cancelled out in multiplication and some columns had never more than one, two, or three terms. The only objection to this method is the guessing how many lines it is going to occupy but this is easily gotten around by leaving the product in the book or carrying it on the edge of another piece of paper until the place is reached for it. Remainders come in quite naturally. You will be surprised how the students take to this method and what a fine mental exercise it is. The mystery is all removed and they do with understanding what is a mere rule by the other method.

This method also has the advantage of leading directly into factoring and furnishes a general method for factoring from which the special methods can be gradually worked out. The students are simply turned loose on a fairly hard exercise in factoring with the suggestion that it is like division, but both of the expressions that were multiplied together are to be found. This furnishes a fine exercise in experiment and observation and the special methods will gradually be evolved. The special rules of multiplication are also met in this way and the connec-

tion between them and the factoring is more direct than by the other methods.

It would not be surprising if the students under such a system did not show the rapidity of advance in technical algebra that they would under the drill system. But I cannot but believe that they are better citizens and in the long run will make better algebricians. In encouraging a person to think it is desirable that there be presented some logical matter to think about in which there is no dispute about the conclusions. This is the value of algebra and geometry in the high school curriculum. Then this matter should be presented in a manner that permits the person not so much to see the truth as to draw conclusions for himself and discover the truth. This takes time and cannot be accomplished when the student is harried by constant drill. What he needs is personal attention to show him just how far his thinking is correct and just where it got off the track and is inconsistent with his other ideas.

AN APPROACH FOR 10B GEOMETRY

Although not expecting to find a royal road to geometry, the teacher of the beginning students is justified in seeking the most desirable path to the 'hinterland,' the path that has the most uniformly gradual assent to the heights. The path here outlined starts near the usual place of vertical angles, but follows angles through multisided polygons and delays the meeting with congruent triangles for nearly a month, in the meantime surmounting such obstacles as the common axioms and many shorter arguments.

The beginning student is assumed to know what he means by a straight line and the opening statement is that if two lines cross, they point in different directions and the difference in direction is an angle. Parallel lines are then defined as lines that point in the same direction. This leads to two conclusions, that parallel lines cannot meet and that if they are crossed by a third line the difference in direction or the angle must be the same at both crossings. Vertical angles are then proved equal, which makes available all the propositions about parallel lines cut by a transversal. The converses are left until the three angles of a triangle are proved equal to 180° and are then made to depend on that fact. Attention is called to the turning of a vehicle

by backing and going forward which is a practical proof that the three angles of a triangle equal 180. A vehicle going around a circle demonstrates the fact that the sum of the exterior angles must be 360 for any number of sides.

These facts permit the use of a long series of problems that demand reasoning with the simple axioms without introducing the longer proofs using congruent triangles.

After congruent triangles have been introduced and the three main cases proved, construction problems are more effective for their exercise than propositions. That is, taking the problem of bisecting an angle, the pupil is more impressed with what are called the given facts if they are true because made true by some other pupil or the teacher. There is more of reality to the problem if one student is asked to bisect an angle that the teacher has drawn and he or another student is asked to prove that what he has done will make the two parts necessarily equal. Other standard constructions can be used in this way and many propositions can be brought in. The various ways of constructing a square is a good exercise for the purpose. It cultivates observation to be ruled out for saying something was done that was not done. The parallelogram can be introduced in this way. The order of the remaining propositions is not important.

This order of introduction of the beginning propositions is preferable to any that I have found. The 'pons asinorum' in the modern teaching of geometry is the argument of the equality of corresponding parts of congruent triangles. This is met ordinarily when all the newness of geometry is fresh upon the student and it is an advantage to put it off for a month. In this month, the teacher, the student and the argumentative method get better acquainted and congruent triangles can be assailed in better formation. The facts about the angles of polygons are a more impressive set of facts to the student than those ordinarily taken up first which are usually no news at all. Also, proving the proposition about the angles of a triangle so early relieves the awkwardness of the proof of several propositions that is so evident in some geometries.

It is inconvenient to try this without a book embodying the idea, but an experienced teacher who is not afraid of getting off the beaten path will find relish in trying any modification of it that may seem best.

A SUPPLEMENTARY PROJECT IN FUNCTIONAL GRAPHS

By J. S. GEORGES
University High School, Chicago

A supplementary project in Mathematics may be used effectively as a means of inspiring those students who display mathematical ability and show special interest in the subject, and of interesting those who take Mathematics as a required subject, but have not developed any particular interest in it. It offers to the latter a glimpse into the more pleasant aspects of the subject not ordinarily included in a text book, and to the former it opens new channels of mathematical interest, creates a desire for the acquisition of further and more extensive mathematical knowledge. The student who voluntarily, or at the suggestion of his instructor, undertakes to do a supplementary project in connection with his classroom work, is offered a splendid opportunity for the formation of the habit of independent thinking, and for acquiring the ability of reading comprehensively mathematical literature. Under proper guidance he lays the foundation for future research work.

Unfortunately the subject of supplementary project is not treated adequately in mathematical literature, in fact, it is hardly mentioned, and often the teacher who feels the need of supplementing his class-room work, whatever be his motive, is handicapped because of this lack of useful material. There is no organized material available. The few text books on general mathematics could hardly be said to furnish easy reading matter to the high school pupil who is just entering the threshold of Algebra or Geometry. And the help he may obtain from mathematical journals along this line is very limited. He is thus thrown upon his own resources. Nevertheless, if he has an adequate training in his subject, enthusiasm in his work, and willingness to promote the welfare of his class in Mathematics, he will find the field of Mathematics rich and varied in useful material for supplementary project work. Many classical problems of the elementary type, original and specially devised problems, or even problems of more advanced types, whose solution is not too difficult by means of the already accumulated information, may be used to advantage.

The subject of graphs, that is, the graphical representation of the functional concept, furnishes a rich field of investigation of the elementary type, which is both interesting and instructive to the student desiring to do supplementary work. The graphical representation of numerical facts and relationships by means of the geometric line segments and curves is important in Mathematics and should be emphasized, and at the same time is also very interesting to the child. The notion is not altogether new to him, having become familiar with it from newspapers and magazines, and the principles underlying this representation are readily understood and are tacitly assumed without much difficulty. It may be used by the beginner who is familiar with the bar graph or the continuous line graph to represent unrelated or related facts. Later on after the study of the equation and the formula, the graph is still of vital importance in the interpretation of the linear, quadratic, or cubic equation. Drawing of pictures is especially interesting to the child, and this geometric picturing of the algebraic notions not only keeps up his interest in the subject, but also impresses upon his mind the true union of these two apparently separate branches of Mathematics. Historically this fusion of Algebra and Geometry is of special interest to the mathematician and the student should not wait until he is in college or doing graduate work to appreciate the full significance of it.

The following project worked out by a junior high school pupil should suggest the type of supplementary work possible in this field. The project was assigned to six students at the end of the unit on Functions, which takes up direct and inverse variation, the functional representation, and evaluation of functions both by substitution and by the synthetic division, and the functional graph. The text book used was Breslich's "Junior Mathematics," Book III. All of the pupils were very enthusiastic in their work and seemed to derive a great deal of pleasure in their ability to trace new and advanced curves. The one reproduced here is selected for its brevity and it is typical of the kind of work handed in.

Subject: The Graphs of Cubic Functions.
Name: Robert E. Ascher, Junior Mathematics, Course III.
Instructor: Mr. Georges.

The graph of the linear function, such as $f(x) = x + 5$, is always a straight line, while that of a quadratic function, such as $f(x) = 2x^2 \times 3$, is a parabola. The graph of the cubic function is generally a curve which crosses the x-axis in three places. In this paper some unusual cubic curves are produced which differ in shape from the general cubic.

In graphing a cubic function usually nine points are needed. First make a table with arbitrary values for x and the corresponding values for $f(x)$. The values of $f(x)$ may be found either by substituting the value of x in the expression, or by using the method of synthetic division. Next select a set of axes and choose a convenient unit to plot the pairs of values. Finally join the points together with a "smooth" curve.

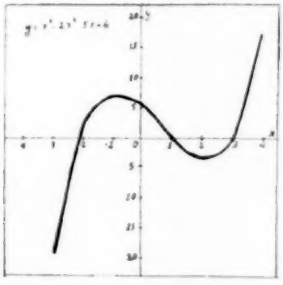

Fig. I

Figure I is the graph of the cubic function $y = x^3 - 2x^2 - 5x + 6$. It intersects the x-axis in three points, at $x = 1$, $x = 3$, and $x = 2$, which are the solutions of the equation $x^3 - 2x^2 - 5x + 6 = 0$.

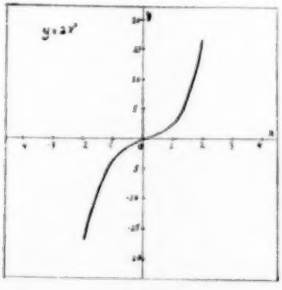

Fig. II

Figure II is the graph of $y = 2x^3$. The curve crosses the x-axis only once, at $x = 0$. The three points are brought together. This curve is called "cubical parabola."

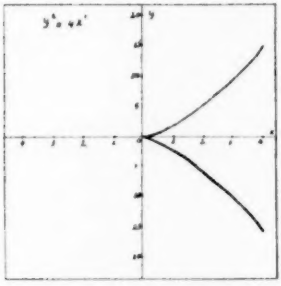

Fig. III

Figure III is the graph of $y^2 = 4x^3$. The graph of this cubic function is of an entirely different shape. All of the curve is to the right of the y-axis. It has a sharp point at the origin. It is called "cuspidal cubic." We can't use negative values for x for we can't extract the square root of negative numbers.

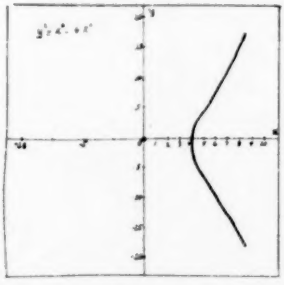

Fig. IV

Figure IV is the graph of $y^2 = x^3 - 4x$. This is still another type of cubic function and there is one place on the graph (at origin) where it does not connect with the rest of the graph at all. The point at the origin is called a "conjugate" point. It intersects the x-axis at $x = 4$.

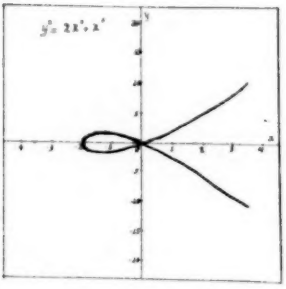

Fig. V

Figure V is the graph of $y^2 = 2x^2 + x^3$. This curve makes a loop with the knot at the origin. It intersects the x-axis at the origin and at $x = -2$. This curve is called "nodal cubic."

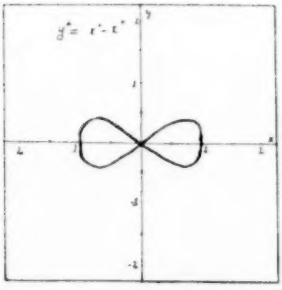

Fig. VI

Figure VI is very interesting. It is the graph of a quartic function $y^2 = x^2 - x^4$. The curve is like the figure eight. It crosses the x-axis at $x = \pm 1$ and the origin.

THE OLDEST ALGORISM IN THE FRENCH LANGUAGE

ANNA ELIZABETH HOUGHTALING AND
FRANCES MARGUERITE CLARKE

Introductory Note

The oldest French Algorism[1] known to be extant is in the Bibliothèque Sainte Geneviève, Paris. There is in the Bibliothèque Nationale another manuscript of a very similar nature but of a later date. Both are of unknown authorship and both are of the thirteenth century.[2]

The former work consists of folios 150, r-151, r;[3] the latter, of folios 154, r- 155, r. Since the two are similar in content, familiarity with the latter facilitates the reading of the former, for the French of the manuscript in the Bibliothèque Nationale is more modern in style. In spite of the fact that the two versions are available, the translation of either is exceedingly difficult in parts, the author being only vaguely familiar with his subject and writing so confusedly that the maze seems in one instance well-nigh hopeless.

There are four divisions of the algorism; (1) a treatment of numbers, (2) a consideration of the six fundamental processes, (3) division by the galley method, and (4) extraction of cube roots. The last two are not very clear and are not wholly correct.

Since no version of the manuscript seems to exist in any modern language (the French being far from the present form), and since the work is historically so important in the history of education as well as the history of arithmetic, the following trans-

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A paper prepared in Professor David Eugene Smith's Practicum in The History of Mathematics, (1924-25).

[1] Mohammed ibn Musa al-Khowarizmi, mathematician of Bagdad, *c* 835 A. D., wrote a treatise on arithmetic. Robert of Chester or Adelard of Bath translated and gave it the title of "Algoritmi de Numero Indorum" (Algoritmi was the nearest Latin spelling of the name al-Khowarizmi, meaning the man from Khwarezm). The various forms of algorism, augorism, etc., derived from Algoritmi, were applied to any study of numbers. Smith, *History of Mathematics*, I, 170; II. 9.

[2] See the Boncompagni *Bulletino* XV, 53, *and the Bibliotheca Mathematica*, IX (3), 60. The date of a manuscript is determined by the handwriting. This is not difficult since the handwriting is distinctive for the period.

[3] When medieval manuscripts were bound, only the folios were numbered on what we would call the right-hand page. The right-hand page of folio 150, for example, is commonly referred to as "150 *r*" (recto), the reverse side of the leaf being referred to as "150 *v*" (verso).

lation has been made. It is as literal as circumstances seem to warrant, but, as already stated, the author himself was so confused in one or two places that no translation can serve to show what he could have meant. The notes are intended to clarify the reading as much as possible, but in some instances offer only a possible solution to the author's thought.

Translation

Algorism is any representation using such symbols as the figures 9, 8, 7, 6, 5, 4, 3, 2, 1. The first is 1, the second 2, the third 3; and so on for the others up to the last which is called the cipher 0.[1]

Any of the numbers written in the unit's place has its original value; in the ten's place, ten times its original value; and in each succeeding place, a value ten times greater than in the place preceding. The cipher has no intrinsic value, but attached to any number it indicates a multiple of that number.

There are three types of numbers,—the digit, the article, and the composite. Numbers from 1 to 9, inclusive, are called digits; 10 and multiples of 10 are called articles; and combinations of articles and digits, such as 11, 12, 13, etc., are termed composites. If a number is a digit, it is written alone; if an article, the cipher is written first[2], and then the article; if a composite, the digit first, and then the article.

There are six parts to algorism: addition, subtraction, duplation, mediation, multiplication, and division. In addition or subtraction, begin at the right; in duplation, multiplication, or division, begin at the left.

In addition, write the greatest number above and the lesser below so that the first figure of the smaller number is beneath the first figure of the greatest number; then write the other figures in similar order. If there are more figures following, add the first two. If the sum is a digit, write it in line with the figures above; if an article, write the zero separately, and then write the article with the figures above in order.

[1] Writing numbers from right to left is due to the Arabic influence.
The name cipher is apparently from the Sanskrit *sunya* (void), through the Arabian *sifr*.
The zero (0) itself appeared in India at least as early as the 9th century and is probably of Hindu origin. Smith, *History of Mathematics*, II, 69, 71.

[2] Beginning at the right.

Division should not result in a quotient exceeding 9.[1] You get one number for each time you divide. Place it beneath the first figure of the number beneath your number; then rewrite the figures of the lower number until the upper is greater than the lower,[2] finally dividing the number above, as you did before. Write the number which indicates how many times the upper number is divided by the lower.[3]

When in division the number above is less than the number beneath,[4] place this number elsewhere.[5] Prove the work by multiplication. If the division is correct, multiplication of the divisor, and addition of the number (remainder) will give the first number.[6] This proves the accuracy of the work.

Any number multiplied by itself [cubically] is a cube. To find the cube [root] of any square[7] number, first write the number, then under the last figure of a thousand write a digit that multiplied by itself cubically, destroys (with respect to this number underneath) the number above as much as possible.[8] Cut it off,[9]

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[1] That is, no figure in the quotient can exceed nine.

[2] This is easily seen in the galley method of dividing, then in use.

[3] To divide 1,140 by 95 write 95 beneath the 1,140.

Ninety-five is contained in 114 one time. Therefore 1 is the first figure in the quotient. Then $1 \times 9 = 9$. This is subtracted from the 11, and the 2 (being the remainder) is placed above the 11. Then $1 \times 5 = 5$. This is subtracted from 24.

The 19 is placed above the 24. Then the 95 is written as shown. Since 95 is contained in 190 twice, 2 is the second figure in the quotient.

$2 \times 9 = 18$ $\quad$ $19 - 18 = 1$

$2 \times 5 = 10$ $\quad$ $10 - 10 = 0$

1140
95
1
29
1140
955
9
9
380
1230
955
9

[4] To divide 1,230 by 95, the work would appear as follows: The remainder is 90. (The number above is less than the number beneath.)

[5] That is, move it one place to the right.

[6] That is, quotient times the divisor, plus the remainder, equals the dividend.

[7] The writer meant "cubic."

[8] For example, to find the cube root of 2197: Write 2197; under 2, the figure at the left, write a number whose cube, when subtracted from 2, gives a number as nearly zero as possible. In this case subtract 1 (the number is really 10). Cubing the 1 and subtracting the result from 2 leaves 1197.

[9] Cut it off means that from $a^3 + 3a^2b + 3ab^2 + b^3$ we tage away a^3.

treble it,[1] and place it two points in advance.[2] Now place the sub-treble below the treble.[3] Then place a new digit in front of the treble[4] which with the sub-treble multiplies the treble; then multiply the sum resulting by itself, and subtract it from the treble.[5] Take the digits by themselves cubically and subtract from the digits.[6] Cut off the digit,[7] treble it and place it two points in advance. Finally take a new digit which with the sub-treble multiplies the treble, and then square this sum, and subtract it from the last. Treble the digit, cube it, and subtract it from the digit. Continue the process thus, to the end. The number so operated on is a cube. The root is the sub-treble and the last digit found; multiply this by itself cubically and you will have again the original number. If there is any remainder the proposed number is not a cube, but you have the greater beneath. Cubing the root and adding the remainder will give the original number. If in trebling the digit no articles are formed, follow the rules of duplation.[8] If work with the trebled number is impossible, place a cipher for the digit; then write it and work as before. If there is only a cipher place a 0 for the digit; subtract it and place a 0. Place a 0 under the digit, and a 0 in line with the sub-treble. If this number whose root is to be taken has but two or three figures, at the right under the first figure, place a digit that multiplied by itself cubically destroys the number above as much as possible. The digit is then the root of the number.

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[1] To treble is to multiply it by 3. The process is really to take $3a^2$.

[2] To place it two points in advance means that, since we really have 3×10^2, instead of 3×1, we have not 3, but 300, the 3 being two more places to the left than in the case of 3.

[3] To place the subtreble below the treble seems to mean that we are to place $3a$ after (below numerically) the $3a^2$.

[4] That is, by dividing find b and place this (the "new digit") before (to the right of, algebraically expressed) the $3a$, giving $3a^2 + 3ab.$, The rest of the statement probably means that he multiplies $3a^2 + 3ab + b^2$ by b, as in algebraic cube root; but it is evident that he does not understand the process.

[5] This would seem to mean that $3ab + 3ab^2 + b^3$ is now subtracted from the cube.

[6] The cube of the first part is subtracted from the given cube.

[7] Apparently he now repeats the process in order to find the third figure.

[8] This seems to mean that duplation may be used in case of trouble in multiplying, but the expression as it stands is meaningless.

The author thus abruptly ends his treatise. Although it is evident that he was hopelessly confused in some parts of this manuscript, especially the latter, it is worthy of consideration, first because of the unique place it holds as the oldest French treatise on algorism; secondly because it offers an interesting example of the early use of one as a number and also the use of the French word "assembler" for our process of addition.

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W. P. Percival,
Macdonald College, Canada.

PERIODICALS

"What New York City is Doing for Dull-Normal Pupils." Ralph E. Pickett, School of Education, New York University. The Chicago Schools Journal, January, 1926. (Read before the National Vocational Guidance Association at Cincinnati, February, 1925.)

The author refers to the work of Seward Park Junior High, Textile High School, and the Manhattan Trade School for Girls, where some definite progress seems to be made in fulfilling the obligation of the schools with respect to those who are usually crowded out through failure and who often become a menace to society.

The dull-normal pupil is roughly defined as one who "is too dull to be called of normal intelligence and yet too normal to be called a moron, an imbecile, or something worse,—one whose I. Q. is probably between 75 and 95." It is for this type of pupil that we have either sapped our course of study of its vitality, or we have assumed that we ought not try to educate him.

While the highest type of academic education is suited to the training of the stronger pupils, as a preparation for leadership in life, the author urges, "Let us disabuse our minds of the tradition that the only real education is academic education, and that only those who can profit by such have a right to be educated. Let us realize our duty to all the children and let us avoid in future the waste of a useless expenditure of millions of dollars and countless precious hours in an attempt to force a mass of children to master that which they cannot master and ought not to be expected to master."

A. D.

The History of Arithmetic by C. L. Karpinski. Rand, McNally & Company, 1925. Pp. 200 + xii. Price $2.00.

The purpose of this book is to provide the teacher of arithmetic with a short, readable discussion of the topics she teaches. The volume begins with a description of systems of numerals other than our own, and the form that arithmetic operations assumed in the case of certain of them. The history of Hindu Arabic

numerals is then introduced, and, after a digression into the subject of notable textbooks in arithmetic, the development of arithmetic operations with these numerals is treated. Subsequent chapters deal with fractions, business arithmetic, terminology, denominate numbers, and the teacher and the teaching of arithmetic.

The work is well illustrated and the author has been clever in calling the reader's attention to salient points in the cuts in the notes that accompany them.

The chapter on textbooks in arithmetic includes a biliography of works printed in America prior to 1800. This is especially useful in that the libraries where these volumes may be found are listed.

The book includes much information that should be of value to the teacher of arithmetic, but it is disappointing in certain respects. Professor Karpinski tends to make assertions without giving the material needed to make his conclusions of the greatest use to his readers. For example, under the caption "History reflected in arithmetic," the author states that the textbooks printed in the Revolutionary and Civil War periods are especially rich in showing the way contemporary conditions were utilized in problem material, but, instead of illustrating this by concrete instances, he passes to a discussion of the fact that the textbooks printed in Colonial America reflect the nationality of the various groups of settlers in the languages in which they appear. This last point is well taken and well substantiated, but the bibliography that follows will be useful to but few, whereas a fuller treatment of the earlier point would be useful to many.

Such a work is informative rather than controversial, yet when Professor Karpinski says that the Roman pound sometimes contained sixteen ounces, it would be interesting to know the source of his information. Where his spelling of proper names departs from that given in such standard works as Professor David Eugene Smith's two volume *History of Mathematics*, it would be desirable to know the reasons that prompted this usage.

VERA SANFORD,
The Lincoln School.

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